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MECHANIC-MATHEMATICAL MODEL FOR INVESTIGATIONS OF THE NATURAL FREQUENCIES AND MODE SHAPES OF THE FREE SPATIAL VIBRATIONS OF WOOD SHAPER AND ITS SPINDLE

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Abstract

An original mechanical - mathematical model of wood shaper and its spindle, developed by the authors, is presented in this work. The model provides the opportunity to explore the free space vibrations of this type of machinery. It takes into account the characteristics in the construction of wood shapers. In this model the wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. The model takes into account the necessary mass, inertia and elastic properties of the elements of the considered system. It includes all necessary geometric parameters of this system. A necessary system of matrix differential equations is compiled and analytical solutions are presented. Numerical solutions can be obtained with their help by using the parameters of a specific machine.

Key words: *wood shapers, free vibrations*

INTRODUCTION

The enhanced requirements for reducing the level of vibration and noise, accompanying the operation of modern woodworking machines, are an essential prerequisite for the expansion and deepening of research of dynamic processes in them [5], [7], [8].

Wood shapers are in the group of woodworking machines with high levels of vibration and noise. The introduction of measures for reduction the level of vibration and noise requires understanding the essence of phenomena characteristic of this machine and its individual elements [2], [6]. It is necessary to conduct concrete studies in which the machine can be considered as a mechanical vibrating system with known characteristics [1], [9], [10]. Some recommendations based on the results of these investigations applicable to the design, manufacture and operation of wood shapers, may be made.

It is known that in the spread of vibrations in the machine vibrations' energy decreases, and some of their characteristics change [3]. The reason for this is the absorption of a part of the energy due to internal losses in the materials of its elements, as well as losses in the construction. The losses in the construction are due to the presence of various joints (screwed joints, riveted joints, welded joints etc.), and the cross-sections changing of the structural elements, as well as means installed for vibration insulation. Suitable vibration

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isolators between the machine and the floor .have to be installed and it is very important for wood shapers. The aim of this is to limit the distribution of machine's vibrations to the environment. Unified elastic elements with known dynamic characteristics are commonly used. Demanding of vibration isolators with suitable qualities is associated with conducting a preliminary study of the vibration behavior of the machine to assess the influence of the elasticity coefficient of the vibration isolators.

Variable loads arise during the operation of the wood shaper on its working tool. They are transmitted to the spindle and by its two bearing units reach to the machine's body. On the other hand, vibrations, generated by other elements of the machine, reach the spindle and the cutter back through bearing units. It is clear that the characteristics of the bearing units (stiffness, damping properties, etc.) are important for the interaction between the spindle with the cutter and the machine's body, and consequently for the work of the whole machine.

The idea that the wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor, derives from the written above. These elastic elements are four vibration isolators between the machine and the floor, and both bearing units of the spindle.

The aim of this work is to build a mechanical - mathematical model of a wood shaper and its spindle, which gives the opportunity for exploration the free space vibrations of this type of machinery. The model refers to wood shapers with lower placement of the spindle. The model renders in account the construction's characteristics of this class of wood shapers. The developed model allows making numerical investigations by using parameters of specific machines.

The kind of wood shapers with lower placement of the spindle that is commonly used in the practice of the forestry industry [4], [5] is examined in the proposed study. Analysis of their construction shows the strong influence of the work spindle on the functioning of the whole machine. Fig. 1 shows the general view, and Fig. 2 – the spindle with its bearing units and fitted cutter.

MECHANICAL-MATHEMATICAL MODEL

In the following discussions, the wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor. These elastic elements are four vibration isolators between the machine and the floor, and the two bearing units of the spindle.

A mechanical - mathematical model of wood shapers with lower spindle is built for studying its free spatial vibrations. The model is shown in Fig. 3.

The following symbols are used:

m_1, m_2 – mass of the wood shaper and its spindle;

$\mathbf{I_1}$, $\mathbf{I_2}$ – inertia moment tensors of the wood shaper and its spindle;

c_{x1i}, c_{y1i}, c_{z1i} , $i = 1, 2, 3, 4$– elastic coefficients of the vibroisolators between the machine and the floor;

c_{x2i}, c_{y2i}, c_{z2i} , $i = 5, 6$– elastic coefficients between the machine and the spindle;

The vector of the generalized coordinates is (Fig. 3)

$$\mathbf{q} = \left[x_1 \; y_1 \; z_1 \; \theta_{x1} \; \theta_{y1} \; \theta_{z1} \; x_2 \; y_2 \; z_2 \; \theta_{x_2} \; \theta_{y_2} \; \theta_{z_2} \right]^{\mathrm{T}} \quad (1)$$

Fig. 1 Wood shaper – general view

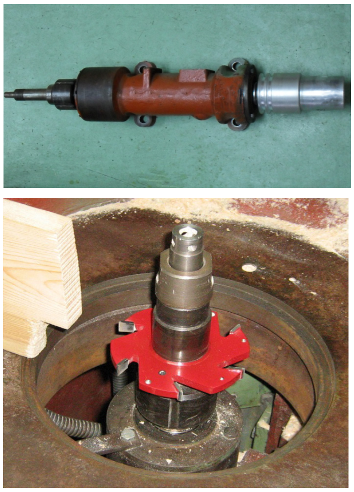

Fig. 2 Spindle with bearing units and with cutter

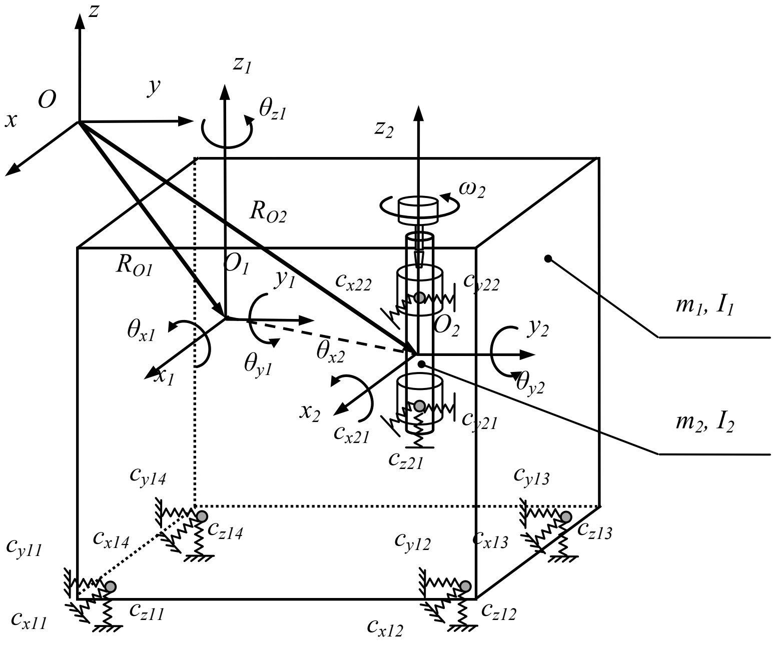

Fig. 3 Mechanic-mathematical model of the wood shaper and its spindle

The matrixes of the transition in small vibrations between the local coordinate systems of the bodies and the reference coordinate system have the form

$$\mathbf{A_i^0} = \begin{bmatrix} 1 & -\theta_{zi} & \theta_{yi} & x_i \\ \theta_{zi} & 1 & -\theta_{xi} & y_i \\ -\theta_{yi} & \theta_{xi} & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad i = 1,2 \qquad (2)$$

The vector of the position of the center of mass of the relevant body is determined with

$$\mathbf{R_{Ci}^0} = \mathbf{A_i^0}.\mathbf{r_{Ci}} = \begin{bmatrix} l_{Cx} + x_i + l_{Cz}.\theta_{yi} - l_{Cy}.\theta_{zi} \\ l_{Cy} + y_i - l_{Cz}.\theta_{xi} + l_{Cx}.\theta_{zi} \\ l_{Cz} + z_i + l_{Cy}.\theta_{xi} - l_{Cx}.\theta_{yi} \\ 1 \end{bmatrix} \qquad i = 1,2 \qquad (3)$$

where $\mathbf{r_{Ci}} = \begin{bmatrix} l_{Cx} & l_{Cy} & l_{Cz} \end{bmatrix}^T$ is the vector of the position of the center of mass in the local coordinate system.

The vector of absolute linear velocity of the center of mass of the respective body is calculated as follows:

$$\mathbf{V_{Ci}^0} = \frac{d\mathbf{R_{Ci}^0}}{dt} = \begin{bmatrix} \dot{x}_i + l_{Cz}.\dot{\theta}_{yi} - l_{Cy}.\dot{\theta}_{zi} \\ \dot{y}_i - l_{Cz}.\dot{\theta}_{xi} + l_{Cx}.\dot{\theta}_{zi} \\ \dot{z}_i + l_{Cy}.\dot{\theta}_{xi} - l_{Cx}.\dot{\theta}_{yi} \\ 0 \end{bmatrix} \qquad i = 1,2 \qquad (4)$$

The vector of absolute angular velocity of the respective body, projected in the local coordinate system, has the form

$$\mathbf{\Omega_i^i} = \begin{bmatrix} \dot{\theta}_{xi} \\ \dot{\theta}_{yi} \\ \dot{\theta}_{zi} \\ 0 \end{bmatrix} \qquad i = 1,2 \qquad (5)$$

The deduction of the kinetic energy and potential energy of the system is convenient to be made with a symbolic method and modern software (Mathematica, MATLAB).

The kinetic energy of the mechanical system is determined with

$$E_K = \sum_{i=1}^{2} \left(\frac{1}{2}.\begin{bmatrix} \dot{R}_{Ci}^T & \dot{\Theta}_i^T \end{bmatrix}^T.\begin{bmatrix} m_{RRi} & \\ & I_{\Theta\Theta i} \end{bmatrix}.\begin{bmatrix} \dot{R}_{Ci} \\ \dot{\Theta}_i \end{bmatrix} \right) \qquad (6)$$

where

$$\mathbf{m}_{\mathbf{RR}i}=\begin{bmatrix} m_i & 0 & 0\\ 0 & m_i & 0\\ 0 & 0 & m_i\end{bmatrix};\ \mathbf{I}_{\mathbf{\Theta\Theta}i}=\begin{bmatrix} I_{xxi} & -I_{xyi} & -I_{xzi}\\ -I_{xyi} & I_{yyi} & -I_{yzi}\\ -I_{xzi} & -I_{yzi} & I_{zzi}\end{bmatrix},$$

$$\dot{\mathbf{R}}_{\mathbf{C}i}=\begin{bmatrix}\dot{x}_{Ci} & \dot{y}_{Ci} & \dot{z}_{Ci}\end{bmatrix}^{\mathrm{T}};\ \dot{\mathbf{\Theta}}_i=\begin{bmatrix}\dot{\theta}_{xi} & \dot{\theta}_{yi} & \dot{\theta}_{zi}\end{bmatrix}^{\mathrm{T}}.$$

Potential energy is defined by

$$E_P=E_{P1}+E_{P2}=\left(\sum_{k=1}^{4}\frac{1}{2}c_k.\left(\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{01}\right)^2+\sum_{k=1}^{2}\frac{1}{2}c_k.\left(\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{12}\right)^2\right)+\sum_{\mathrm{i}=1}^{2}\mathrm{E}_{P\mathrm{Gi}} \qquad (7)$$

where

$$\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{01}=\mathbf{R}_{\mathbf{1}}+\mathbf{U}_{\mathbf{1}}^{\mathbf{0}}.\mathbf{r}_{\mathbf{k}}^{01}-\mathbf{r}_{\mathbf{k}}^{01},$$

$$\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{12}=\left(\mathbf{R}_{\mathbf{1}}+\mathbf{U}_{\mathbf{1}}^{\mathbf{0}}.\mathbf{r}_{\mathbf{k}}^{12}-\mathbf{r}_{\mathbf{k}}^{12}\right)-\left(\mathbf{R}_{\mathbf{2}}+\mathbf{U}_{\mathbf{2}}^{\mathbf{0}}.\mathbf{r}_{\mathbf{k}}^{21}-\mathbf{r}_{\mathbf{k}}^{21}\right),$$

$\mathbf{R}_{\mathbf{i}}=\begin{bmatrix}x_i & y_i & z_i\end{bmatrix}^{\mathrm{T}}$ $\quad i=1,2$ - vector of the position of the beginning of the mobile (related with the body) coordinate system relative to the fixed coordinate system,

$\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{01}$ - the deformation of the elastic elements between the base (marked conditionally with "0") and the body 1,

$\boldsymbol{\delta}\mathbf{r}_{\mathbf{k}}^{12}$ - the elastic deformation of the elements between the two bodies.

The differential equations which describe the free vibrations are deduced by using the Lagrange's method. This method provides the best opportunities.

$$\frac{d}{dt}\left(\frac{\partial E_K}{\partial \dot{q}}\right)-\frac{\partial E_K}{\partial q}+\frac{\partial E_P}{\partial q}=0 \qquad (8)$$

where E_K and E_P are respectively the kinetic and the potential energy of the systems.

The obtained system of parametric differential equations, which describes the free vibrations of the mechanical system, is

$$\mathbf{M}.\ddot{\mathbf{q}}+\mathbf{C}.\mathbf{q}=0 \qquad (9)$$

The matrix, which characterizes the mass-inertial properties **M** and the elastic properties **C** of the mechanical system, is

$$\mathbf{M}=\begin{bmatrix}a_{ij}\end{bmatrix},\qquad a_{ij}=\frac{\partial^2 E_K}{\partial \dot{q}_i.\partial \dot{q}_j}, \qquad (10)$$

$$\mathbf{C} = \left[c_{ij}\right], \qquad c_{ij} = \frac{\partial^2 E_P}{\partial q_i . \partial q_j}. \tag{11}$$

Particular solutions to the system of the differential equations (9) are searched as

$$q_r = h_r . \sin\left(\omega_r . t + \varphi\right) \tag{12}$$

where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is the initial phase.

After differentiation of (12) and substituting in (9), a system of linear algebraic equations is obtained. In the matrix description they are:

$$\left|\mathbf{C} - \omega^2 . \mathbf{M}\right| . \mathbf{V} = 0 \tag{13}$$

To determine the natural frequencies and the mode shapes, the task about finding the natural values and the natural vectors of the equations need to be solved(13). The satisfaction of the equations (13) requires the following

$$\det\left|\mathbf{C} - \omega^2 . \mathbf{M}\right| = 0 \tag{14}$$

The roots of the characteristic equation determine the natural frequencies. The natural frequencies form the matrix of the natural values are

$$\omega = diag\left\lfloor \omega_{r,r} \right\rfloor, \qquad i = 1,\ 2, ..., 12 \tag{15}$$

and in [*Hz*]

$$\mathrm{f_r} = \frac{\omega_{\mathrm{r,r}}}{2\pi} \quad Hz \tag{16}$$

The natural values of the system (13) determine the natural vectors of the mechanical system.

A natural vector $\mathbf{v_r}$, which gives correlation between amplitudes of the vibrations, corresponds to each natural frequency ω_r. The vector's components define the matrix of the natural vectors (modal matrix) of the system (13) that is

$$\mathbf{V} = \left\lfloor \mathbf{v}_{r,j} \right\rfloor_{12\times12} \tag{17}$$

where

$$\mathbf{v_r} = \left\lfloor \mathrm{v}_{r,1} \quad \mathrm{v}_{r,2} \quad \mathrm{v}_{r,3} \quad \mathrm{v}_{r,4} \quad \mathrm{v}_{r,5} \quad \mathrm{v}_{r,6} \quad \mathrm{v}_{r,7} \quad \mathrm{v}_{r,8} \quad \mathrm{v}_{r,9} \quad \mathrm{v}_{r,10} \quad \mathrm{v}_{r,11} \quad \mathrm{v}_{r,12} \right\rfloor$$

is the natural mode vector on the generalized coordinate for r^{-th} natural frequency.

CONCLUSION

An original mechanical - mathematical model of wood shaper and its spindle is developed in this study. The model provides the opportunity for exploration the free space vibrations of this type of machinery. It takes into account the characteristics in the construction of wood shapers. The wood shaper and its spindle are regarded as rigid bodies, which are connected by elastic elements with each other and with the motionless floor in this model. These elastic elements are four vibration isolators between the machine and the floor, and both bearing units of the spindle. The model takes into account the necessary mass, inertia and elastic properties of the elements of the examined mechanical system. It includes all necessary geometric parameters of this system. A required system of matrix differential equations is composed and analytical solutions are presented. The next part of this work presents obtained on their base numerical solutions by using the parameters of a specific machine.

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