TORSIONAL VIBRATIONS IN THE SAW UNIT OF A KIND OF CIRCULAR SAW. NUMERICAL INVESTIGATIONS OF THE NATURAL FREQUENCIES AND MODE SHAPE

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Abstract

A numerical investigation of the natural frequencies and mode shapes of the circular's saw unit is presented in this study. The research is done on the base of an adequate mechanic-mathematical model for investigation of free torsional vibrations of a circular saw developed by the authors. The model presents features in the construction of a kind of circular saws. As a result this study allows the determination of the resonant work regimes. The determination of these regimes is important for introduction of adequate measures which can guarantee their using. The results of the investigation can be used as a base for making some recommendations concerning the increase of reliability of the machine as well as the accuracy and quality of the production.

Key words: *Circular saws, modeling, torsional vibrations*

INTRODUCTION

The decrease of the level of the vibrations and the noise during the work of modern circular machines is one of the main problems, which is imposed of high requirements to the parameters of the technical equipment in the wood industry. It is necessary to study the essence of processes that are typical for the machine and its elements to find out reasons for the appearance and increase of the vibrations and the noise. It is also necessary to make some investigations which consider the machine as a mechanical vibration system with some characteristics [6], [11].

The restriction of the vibrations and the noise level demands to be formulated concrete ways and methods for influence on the vibration system. It leads to introduction of accurate requirements connected with the construction and the work of its elements. Therefore, formulation and analysis of the equations which describe the vibrations of the elements of the woodworking machines are very important. These equations can be used as a base for investigations and giving some recommendations to the construction and the way of its work [4].

The kind of circular machines which are wide used in practice are investigated in this study [5], [10]. Figure 1 shows a circular machine and a scheme of this circular machine. The electric motor is presented by *1*, *2* is the belt gear, *3* – the work table, *4* – the main shaft, *5* – the machine's body, *6* – the carriage, *7* – the treated detail, *8* – the circular saw with the flanges and the nut of the main shaft.

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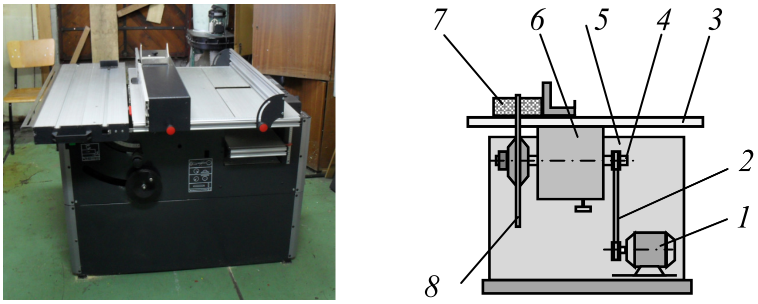

Fig. 1 Circular Machine
1 – Electric Motor, *2* – Belt Gear, *3* – Work Table, *4* – Main Shaft, *5* – Machine's Body, *6* – Carriage, *7* – Treated Detail, *8* – Circular Saw.

Figure 2 shows the unit saw of the circular machine which is investigated in this study.

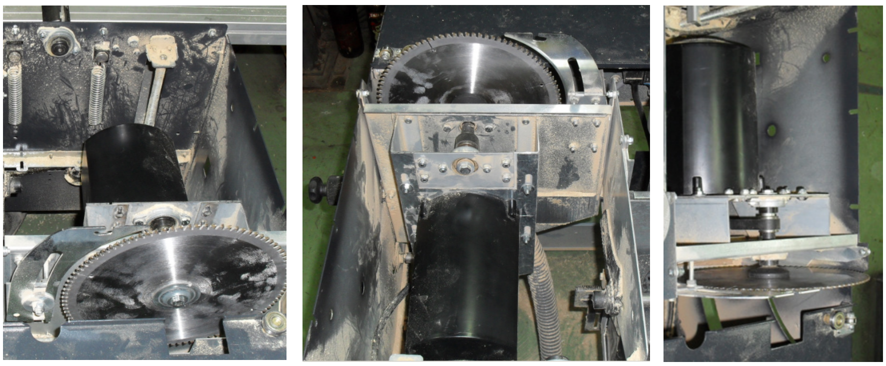

Fig. 2 Unit saw of the circular machine

As every mechanical vibration system, the vibration characteristics of the circular saw have their natural vibration frequencies. When the frequencies of the external influences which cause vibrations are equal to a frequency of their natural frequencies appears the phenomenon "resonance". Resonance regimes can lead to significant increase of vibration amplitudes. Significant vibration amplitudes change the normal work regimes of the machine and damage the accuracy and quality of the production [7], [13]. Extra stress, which is caused by increase of vibration amplitudes, sometimes can reach such values that can damage or even destroy some machine's elements [8].

Principally resonant effects are unwilling. They can be avoided by a suitable selection of parameters of the circular saw and other details and units of the machine, as well as the work regimes [1], [9]. It is necessary to make in advance an evaluation of the resonant danger when the circular machine is designed and dimensioned. It does not allow danger work regimes during the operation exploitation. To solve this problem it is a must to study the natural frequencies. The changes in the construction or in the work regimes are advisable if the resonant danger is available.

The aim of this study is to make a numerical investigation of the natural frequencies and mode shapes of the circular's saw unit. The investigation is done on the base of an adequate mechanic-mathematical model for investigation of free torsional vibrations of a circular saw developed by the authors [12]. The model presents features in the construction of a kind of circular saws. Some recommendations concerning the prevention from the resonant work regimes can be made on the base of this study. It is connected with the increase of reliability of the machine as well as with the accuracy and quality of the production.

MECHANIC-MATHEMATICAL MODEL

An original mechanic-mathematical model for investigation of the dynamical processes and vibrations in the saw unit of a kind of circular saws is built. It is shown on the fig. 3. This model includes four discrete mass connected with three massless elastic elements. φ_i, $i = 1, 2, 3, 4$ are the angles of the rotation of the corresponding rotor. The elasticity coefficients of the electric motor's shaft, the belt and the main shaft are taken into account. The elasticity angular coefficient of the electric motor's shaft is marked with c_1, and this one of the main shaft – with c_3 (*N.m/rad*). The elasticity linear coefficients of the two parts of the belt between the belt puller are c_{23} and c_{32} (*N/m*).

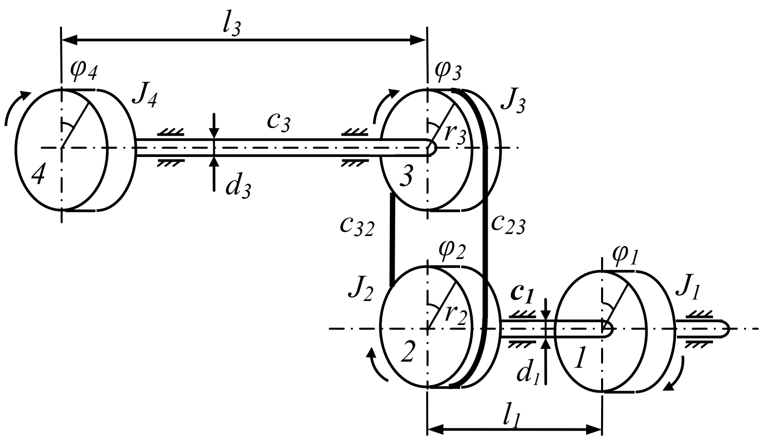

Fig. 3 Dynamical model

The reduced mass inertia moments ($kg{\cdot}m^2$) render in account:
J_1 – the mass inertia moment of the electric motor's rotor;
J_2 – the mass inertia moment of the belt puller on the electric motor's shaft;
J_3 – the mass inertia moment of the belt puller on the main shaft;
J_4 – the mass inertia moment of the circular saw.
Here are some symbols:
d_1, d_3 – diameters of the electric motor's shaft and main shaft (m);
l_1, l_3 – computing length of the electric motor's shaft and main shaft (m);
r_2, r_3 – radius of the belt pullers on the electric motor's shaft and main shaft (m);
G – modulus of shearing.

The investigation of the vibrations of the circular's unit saw requires formulation and solution of the differential equations which describe these processes. Therefore, it is used the matrix mechanics [2], [3].

The mechanic-mathematical model is done by using the applied engineer program (Mathematica). It is developed an algorithm for formulation of the matrixes which describe the mass-inertial properties and the elastic properties of the mechanical system. The differential equations which describe the free vibrations are deduced by using the Lagrange's method.

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right)-\frac{\partial T}{\partial q}+\frac{\partial L}{\partial q}=0\,, \tag{1}$$

where q_i are the generalized coordinates, T and L are respectively the kinetic and the potential energy of the multibody systems.

The vector of the generalized coordinates is

$$\mathbf{q}=\begin{bmatrix}\varphi_1 & \varphi_2 & \varphi_3 & \varphi_4\end{bmatrix}^T. \tag{2}$$

The kinetic energy of the mechanical system is obtained as a sum of the kinetic energy of the four basic bodies (the electric motor's rotor, the belt puller on the electric motor's shaft, the belt puller on the main shaft, circular saw)

$$T=\frac{1}{2}J_1.\dot{\varphi}_1^{\,2}+\frac{1}{2}J_2.\dot{\varphi}_2^{\,2}+\frac{1}{2}J_3.\dot{\varphi}_3^{\,2}+\frac{1}{2}J_4.\dot{\varphi}_4^{\,2}. \tag{3}$$

The potential energy of the mechanical system is obtained as a sum of the potential energies received from the deformations of the electric motor's shaft, the belt and the main shaft.

$$\begin{aligned}L=&\frac{1}{2}c_1.(\varphi_1-\varphi_2)^2+\frac{1}{2}c_{23}.(r_2.\varphi_2-r_3.\varphi_3)^2+\\&+\frac{1}{2}c_{32}.(r_3.\varphi_3-r_2.\varphi_2)^2+\frac{1}{2}c_3.(\varphi_3-\varphi_4)^2\end{aligned}. \tag{4}$$

The system of parametric differential equations which describe the free torsional vibrations of the circular's saw unit are

$$\mathbf{M}.\ddot{\mathbf{q}}+\mathbf{C}.\mathbf{q}=0\ . \tag{5}$$

The matrix, which characterizes the mass-inertial properties of the mechanical system, is

$$\mathbf{M}=\left[a_{ij}\right],\qquad a_{ij}=\frac{\partial^2 T}{\partial \dot{q}_i.\partial \dot{q}_j},$$

$$\mathbf{M}=\begin{bmatrix}J_1 & 0 & 0 & 0\\ 0 & J_2 & 0 & 0\\ 0 & 0 & J_3 & 0\\ 0 & 0 & 0 & J_4\end{bmatrix}=\begin{bmatrix}0{,}001018 & 0 & 0 & 0\\ 0 & 0{,}000018 & 0 & 0\\ 0 & 0 & 0{,}000003 & 0\\ 0 & 0 & 0 & 0{,}011325\end{bmatrix}. \tag{6}$$

The matrix, which characterizes the elastic properties of the mechanical system, is

$$\mathbf{C} = \left[c_{ij}\right], \qquad c_{ij} = \frac{\partial^2 L}{\partial q_i . \partial q_j},$$

$$\mathbf{C} = \begin{bmatrix} c_1 & -c_1 & 0 & 0 \\ -c_1 & c_1 + c_{23}.r_2^2 + c_{32}.r_2^2 & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & 0 \\ 0 & -c_{23}.r_2.r_3 - c_{32}.r_2.r_3 & c_3 + c_{23}.r_3^2 + c_{32}.r_3^2 & -c_3 \\ 0 & 0 & -c_3 & c_3 \end{bmatrix} = \tag{7}$$

$$= \begin{bmatrix} 13004{,}7 & -13004{,}7 & 0 & 0 \\ -13004{,}7 & 13260{,}7 & -144 & 0 \\ 0 & -144 & 9304{,}16 & -9223{,}16 \\ 0 & 0 & -9223{,}16 & 9223{,}16 \end{bmatrix}.$$

Particular solutions to the system of the differential equations (5) are searched as:

$$q_r = h_r . sin\left(\omega_r . t + \varphi\right), \tag{8}$$

where h_r is the amplitude of the small vibration on the generalized coordinate q_r with natural frequency ω_r, and φ is the initial phase.

After differentiation of (8) and substituting in (5) it obtains a system of linear algebraic equations. In the matrix description they are

$$\left|\mathbf{C} - \omega^2 . \mathbf{M}\right| . \mathbf{V} = 0 . \tag{9}$$

To determine the natural frequencies and the mode shapes, it is necessary to solve the task about finding the natural values and the natural vectors of the equations (9).
The satisfaction of the equations (9) requires the following

$$det\left|\mathbf{C} - \omega^2 . \mathbf{M}\right| = 0 . \tag{10}$$

The roots of the characteristics equation determine the natural frequencies. The natural frequencies form the matrix of the natural values. They are

$$\omega = diag\left[\omega_{r,r}\right], \qquad i = 1,\ 2, ..., 4 . \tag{11}$$

The natural frequencies [*Hz*] are determinated by (11)

$$f_r = \frac{\omega_{r,r}}{2\pi} \quad Hz . \tag{12}$$

The natural values of the system (10) determine the natural vectors of the mechanical system.

A natural vector $\mathbf{v_r}$, which gives correlation between amplitudes of the vibrations, corresponds to every natural frequency ω_r. The vector's components define the matrix of the natural vectors (modal matrix) of the system (9) that is

$$\mathbf{V} = \lfloor \mathbf{v}_{r,j} \rfloor_{4\times 4} \, , \qquad (13)$$

where

$$\mathbf{v_r} = \begin{bmatrix} v_{r,1} & v_{r,2} & v_{r,3} & v_{r,4} \end{bmatrix} \, ,$$

is the natural mode vector on the generalized coordinate for r^{-th} natural frequency.

RESULTS

The elements of the circular's saw unit are modeled by using the applied engineer program Solid Works [14]. The mass, elastic and geometrical characteristics are shown in the table 1.

J_1 – inertia moment of the electric motor's rotor ($kg\cdot m^2$)	0, 001018
J_2 – inertia moment of the belt puller *2* ($kg\cdot m^2$)	0, 000018
J_3 – inertia moment of the belt puller *3* ($kg\cdot m^2$)	0, 000003
J_4 – inertia moment of the circular saw ($kg\cdot m^2$)	0, 011325
c_1 – stiffness of the electric motor's shaft (Nm/rad)	13004, 7·
c_2 – stiffness of the main shaft (Nm/rad)	9223,16
c_{23} – stiffness of the belt (N/m)	$5.\cdot 10^5$
c_{32} – stiffness of the belt (N/m)	$5.\cdot 10^5$
d_1 – diameter of the electric motor's shaft (m)	0,017
d_3 – diameter of the main shaft (m)	0,017
r_2 – radius of the belt puller *2* (m)	0,016
r_3 – radius of the belt puller *3* (m)	0,009
l_1 – distance between the belt puller *2* and the electric motor (m)	0,050
l_3 – distance between the circular saw and the belt puller *3* (m)	0,0705

Table 1

The calculations are done with help of the applied engineer program Mathematica [15]. The natural frequencies [s^{-1}] (and in [min^{-1}]) are

55699,2; 27436,2; 556,524; 0; (531884; 261357; 4750; 0).

The calculated natural frequencies [*Hz*] and mode shapes of the mechanism's torsional vibrations are illustrated on fig. 4.

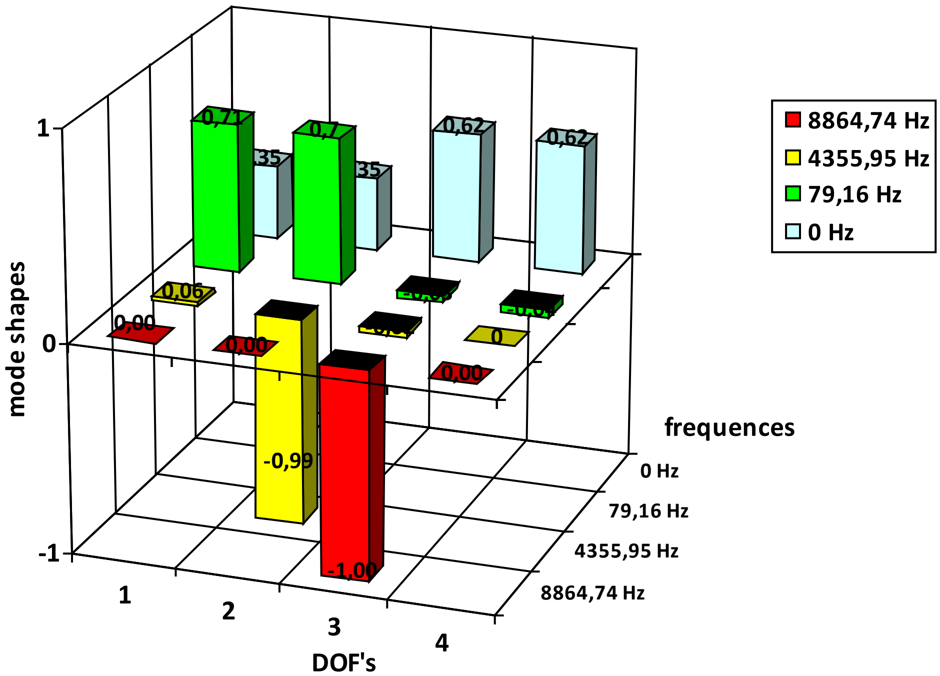

Fig.4 Mode shapes

CONCLUSION

A numerical investigation of the natural frequencies and mode shapes of the circular's saw unit is presented in this study. The investigation is done on the base of an adequate mechanic-mathematical model for investigation of free torsional vibrations of a circular saw developed by the authors. The model presents features in the construction of a kind of circular saws. The results of this study allow making some recommendations to avoid the resonant work regimes. Thus the results can be used as a base for increase of reliability of the machine as well as the accuracy and quality of the production.

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