CUTTING POWER FORECASTING WHILE WOOD SAWING: FRACTURE MECHANICS APPROACH AND AXELSSON’S MODEL COMPARISON

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Abstract
In the classical approach, energetic effects (cutting forces and cutting power) of wood sawing process are generally calculated on the basis of the specific cutting resistance, which is in the case of wood cutting the function of more or less important factors. On the other hand, cutting forces (power) could be considered from a point of view of modern fracture mechanics. Another way is to forecast cutting power consumption on the basis of existing in the literature empirical models. The prognostication of energetic effects has been done for a real sawing pattern applied in a sawmill, in which mainly pine wood is sawn into lumber. The goal of this study was to compare predictions of cutting powers for the circular sawing machine with one circular saw blade with the use of models which base on modern fracture mechanics and a predictive empirical model proposed by Axelssons.

Key words: circular sawing machine, cutting power, fracture mechanics, Axelsson’s model

INTRODUCTION

In the classical approach, energetic effects (cutting forces and cutting power – more interesting from energetic point of view) of wood sawing process are generally calculated on the basis of the specific cutting resistance $k_c$ (cutting force per unit area of cut) (Manžos 1974, Orlicz 1988, Orlowski et al. 2012a, Siklienka and Kminiak 2013). The latter has been confirmed in the latest review paper by Naylor and Hackney (2013), in which sawing became the focus of interest.

In this approach (Böllinghaus et al. 2009), in the chip formation zone the cutting energy applied $E_c$ is converted. The energy converted during machining one unit of volume is:

$$ e_c = \frac{E_c}{V_w} $$

(1)

where $e_c$ is the specific energy and $V_w$ is the volume of material removed. From the specific energy $e_c$ the specific cutting resistance $k_c$ can be derived as a characteristic value for calculating the cutting force $F_c$.
where $P_c$ is the cutting power, $Q_w$ is the material removal rate, $A$ is the undeformed chip cross section, $v_c$ is cutting speed and $F_c$ is the cutting force. In case of wood sawing in the referred literature values of the specific cutting resistance $k_c$ for pine (*Pinus sylvestris* L.) are provided. For other species and cutting conditions recommended coefficients should be applied. Many of those traditional models are empirical and based upon limited information employing blades having standard thickness kerfs. Moreover, for each type of sawing kinematics, different values of specific cutting resistance $k_c$ have to be applied (Manžos 1974; Orlicz 1988; Siklienka and Kminiak 2013).

The methods of cutting power determination during sawing with circular saw blades became the focus of interest in some newer works as follows: Sitkei (2013) studied similarities of the energy requirement of saws (frame saws, bandsaws and circular saws), Cristóvão et al. (2013a, b) compared the industrial results of cutting power measurements with the original Axelsson’s model (1993) outcomes. In the models for a circular sawing machine kinematics described in works by Orlowski et al. (2012b, 2013), similarly to metal milling (Kim and Ehmann 1993), the sum of all uncut chip thicknesses of the simultaneously teeth engaged represented the mean uncut chip thickness. Nevertheless, in reality the instantaneous uncut chip thickness at a certain location of the cutting tooth changes its value (Fig. 1). Hence, Orlowski and Ochrymiuk (2013) have converted the model described in the paper (Orlowski et al. 2013) into a new model in which besides variable uncut chip thicknesses additionally variable values of fracture toughness and shear yield stresses according to the tooth position in relation to the grains were taken into account.

The goal of this study was to compare predictions of cutting powers for the circular sawing machine with one circular saw blade in the case of dry pine sawing obtained with the use of two models which seem to be a kind of universal ones:
- the cutting model that include work of separation in addition to plasticity and friction in which teeth engaged represented the mean uncut chip thickness (Orlowski et al. 2013);
- above mentioned a new developed model (Orlowski and Ochrymiuk 2013),
- a predictive model proposed by Axelsson (Axelsson et al. 1993) for two approaches: the first when teeth engaged represented the mean uncut chip thickness, and the second while variable uncut chip thicknesses and the tooth position in relation to the grains were taken into account.

## THEORETICAL BACKGROUND

### Cutting power model with elements of modern fracture mechanics incorporated

In the case of circular sawing, identically as in analytical models for milling (Ammar *et al.* 2009, Budak 2006), the instantaneous uncut chip thickness $h_j(\varphi)$ at a certain location of the cutting edge can be approximated as follows:

$$h_j(\varphi) = f_z \sin \varphi_j$$

where: $f_z$ is feed per tooth, $\varphi_j$ is the angular position of the $j$-th tooth (immersion angle), and its value changes as follows:

$$\varphi_j = \varphi + (j - 1)\varphi_p$$

where $j = 1, \ldots, z$ (4)
is the pitch angle defined as $\varphi_p = \frac{2\pi}{z}$, and $z$ is number of teeth.

If $\varphi_{en} \leq \varphi_f \leq \varphi_{ex}$, then it has a value, otherwise it is null. $\varphi_{en}$ is an angle of teeth entrance which is given by $\varphi_{en} = \arccos \frac{2(H_p + a)}{D}$ (when the tool tooth gets into the workpiece for machining), and $\varphi_{ex}$ is an exit angle (the tooth of the saw blade gets out of the workpiece) which can be determined as $\varphi_{ex} = \arccos \frac{2a}{D}$. $D$ is a diameter of the circular saw blade. In the case of cutting with circular saw blades the cutting power a new developed macro-mechanic model, which is based on the model proposed initially in work (Orlowski et al. 2013), can be expressed as:

$$P_{cf}(\varphi) = v_c S_i \left[ \frac{\tau_{\parallel \perp - J}(\varphi) \cdot \gamma_f(\varphi)}{Q_{\text{shear}, J}(\varphi)} h_j(\varphi) + \frac{R_{\parallel \perp - J}(\varphi)}{Q_{\text{shear}, J}(\varphi)} \right]$$  \hspace{1cm} (5)

where: $v_c$ is cutting speed, $S_i$ is the kerf (overall set), $\tau_{\parallel \perp - J}(\varphi)$ is the shear yield stress, $\gamma_f(\varphi)$ is the shear strain along the shear plane, $\Phi_g(\varphi)$ is the shear angle which defines the orientation of the shear plane with respect to cut surface, $R_{\parallel \perp - J}(\varphi)$ is specific work of surface separation/formation (fracture toughness), and $Q_{\text{shear}, J}(\varphi)$ is the friction correction (Atkins 2003, Orlowski et al. 2013, Orlowski and Ochrymiuk 2013). The shear angle $\Phi_g(\varphi)$ is material dependent (Atkins 2003) and can be computed numerically (Atkins 2003, Orlowski et al. 2013, Orlowski and Ochrymiuk 2013) from the equation:

$$= -\left\{ \cot \Phi_g(\varphi) + \tan \Phi_g(\varphi) - \gamma_f \right\} \left[ \frac{\sin \beta_f}{\cos(\beta_f - \gamma_f) \cdot \cos[\Phi_g(\varphi) - \gamma_f]} \right]$$

$$= -\left\{ \cot \Phi_g(\varphi) + \tan \Phi_g(\varphi) - \gamma_f \right\} \left[ \frac{\sin \beta_f}{\cos(\beta_f - \gamma_f) \cdot \cos[\Phi_g(\varphi) - \gamma_f]} \right]$$

Figure 1. Sawing kinematics on circular sawing machine: $H_p$ workpiece height (depth of cut), $a$ position of the workpiece, $\varphi_f$ angular tooth position, $\Phi_g$ an angle between grains and the cutting speed direction (Orlowski and Ochrymiuk 2013)
The parameter \( Z = \frac{R_{\perp,j}(\phi)}{\tau_{\perp,j} - h_j} \) makes \( \Phi_j(\phi) \) material dependent.

Taking into account the position of the cutting edge in relation to the grains, for indirect positions of the cutting edge fracture toughness \( R_{\perp,j}(\phi) \) and the shear yield stress \( \tau_{\perp,j}(\phi) \) may be calculated from formulae:

\[
R_{\perp,j}(\phi) = R_{\|} \cos^2 \phi_j + R_{\perp} \sin^2 \phi_j, \quad \tau_{\perp,j}(\phi) = \tau_{\|} \cos^2 \phi_j + \tau_{\perp} \sin^2 \phi_j
\]  

In the case in which teeth engaged have represented the mean uncut chip thickness the cutting power is calculated from Equation (5) for an average uncut chip thickness \( \bar{h} \) given by \( \bar{h} = f_c \sin \phi \), and an average angle of tooth contact with a workpiece \( \phi \) calculated from \( \phi = \frac{\phi_{en} + \phi_{ex}}{2} \). The Equation (5) must be multiplied by \( z_a = \frac{\phi_{ex} - \phi_{en}}{\phi_p} \) which is the number of teeth being in contact with the kerf (average). This approach is described in details in the work by Orlowski et al. (2013), and the average cutting power \( P_c \) is the result of this kind of computation. It is proposed to call this model as FM_A (Fracture Mechanics_Average).

For the model FM_V (Fracture Mechanics_Variable) in which besides variable uncut chip thicknesses additionally variable values of fracture toughness and shear yield stresses according to the tooth position in relation to the grains are taken into account, an average or RMS (root mean square) values of power can be determined after one full revolution of the tool, i.e. \( \phi \) = 0–360° is simulated (Budak 2005). Thus, the total cutting power can then be computed as:

\[
P_c(\phi) = \sum_{j=1}^{j_{\phi,\max}} \frac{\phi_{en} + \phi_{ex}}{2} P_{vj}(\phi)
\]

Axelsson’s predictive model

Axelsson et al. (1993) established a predictive model using multivariate methods such as multiple linear regression and partial least squares regression for Scots pine, of which the range of application has been extended by Cristóvão et al. (2013a, b). The original model has been multiplied by the ratio of the kerf \( S_t \) to the kerf value \( S_t = 4.25 \text{ mm} \) The latter was applied in Axelsson’s experiments. Hence, the extended Axelsson’s model is given by:

\[
F_c = \left( -7.37 + A_1 + 15.61 \Phi_{G,\text{wc}} - 2.6 \Phi_{G,\text{wc}}^3 + 1.31 \rho_{CE} + 0.2 \nu_c + A_2 \right) \frac{S_t}{4.25}
\]  

where: an angle between grains and the cutting speed direction \( \Phi_{G,\text{wc}} = \phi_j \) in radians, \( A_1 = (0.38 \rho - 224.5 \gamma_f) h_j(\phi) \), and \( A_2 = (0.3 \cdot \Phi_{G,\text{wc}} - 0.01 T) \cdot MC \cdot \gamma_f \) is a rake angle (radian), \( \rho_{CE} \) is a cutting edge radius in \( \mu \text{m} \), \( \rho \) is a wood density \( (\text{kg} \cdot \text{m}^{-3}) \), \( MC \) is moisture content (\%), and \( T \) is wood temperature (°C), \( h_j \) is given by Eq. 3.

In order to compute with this model cutting power the Equation 9 must be multiplied additionally by cutting speed \( \nu_c \). As it was described above. In the simplest case calculations may be done for the mean uncut chip thickness \( \bar{h} \) and \( P_c \) the result (AM_A – Axelsson’s Model_Average). On the other hand, if variable uncut chip thicknesses is taken
into account, an average or RMS (root mean square) values of cutting power can be determined after one full revolution of the tool (AM_V – Axelsson’s Model_Variable).

MATERIAL AND METHODS

Predictions of cutting powers have been made for the case of sawing on the circular sawing machine (HVS R200, f. HewSaw), which is used in Polish sawmills. The basic sawing machine data and cutting parameters for which computations were done are shown in Table 1. Computations were carried out in each case for one saw blade (in the diameter of 350 mm), with: a new analytical model described in this paper (FM_V), at the feed speed $v_f$ = 70; 110; and 153 m·min$^{-1}$ (the latter is usually applied at the sawmill), and the model FM_A presented by Orlowski et al. (2013), in which calculations are done for the mean uncut chip thicknesses for the range of the machine tool feed speeds ($v_f$ = 60–200 m·min$^{-1}$). Additionally, calculations were performed with the Axelsson’s model AM_V for variable uncut chip thicknesses, and also for the mean uncut chip thicknesses (AM_A), for the same data applied.

Table 1. Tool and machine tool data (Orlowski and Ochrymiuk 2013)

<table>
<thead>
<tr>
<th>$H_p$ [mm]</th>
<th>$n_{sb}$ [mm]</th>
<th>$S_1$ [mm]</th>
<th>$v_c$ [ms$^{-1}$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>6</td>
<td>3.6</td>
<td>63.95</td>
</tr>
<tr>
<td>$\gamma$ [º]</td>
<td>$z$ [-]</td>
<td>$v_f$ [m·min$^{-1}$] ([ms$^{-1}$])</td>
<td>$f_c$ [mm]</td>
</tr>
<tr>
<td>25</td>
<td>30</td>
<td>60–200 (1 – 3.33)</td>
<td>0.57–1.91</td>
</tr>
<tr>
<td>h [mm]</td>
<td>$v_f$ [m·min$^{-1}$] ([ms$^{-1}$]) usually applied</td>
<td>$f_c$ [mm] applied</td>
<td>h [mm] applied</td>
</tr>
<tr>
<td>0.273–0.913</td>
<td>70 (1.17)</td>
<td>0.67</td>
<td>0.32</td>
</tr>
<tr>
<td>$P_{EM}$ [kW]</td>
<td>$P_i$ [kW]</td>
<td>$P_{ca}$ ($P_{ca}^1$) [kW]</td>
<td>$\varphi_p$ [º]</td>
</tr>
<tr>
<td>90</td>
<td>14</td>
<td>64.6 (10.77)</td>
<td>12</td>
</tr>
</tbody>
</table>

Legend: $P_{EM}$ – electric motor power, $P_i$ – idling power, $P_{ca}$, ($P_{ca}^1$) – available cutting power in the cutting zone (available cutting power per one saw blade), $n_{sb}$ – number of saw blades

The raw material was pine wood (Pinus sylvestris L.) of depth of cut equal to $H_p$, at moisture content MC 8.5–12%, derived from the Baltic Natural Forest Region in Poland (Region A). The value of friction coefficient $\mu = 0.6$ for dry pine wood was taken according to Glass and Zelinka (2010).

Table 2. Raw material data (Orlowski et al. 2012b)

<table>
<thead>
<tr>
<th>Region</th>
<th>$\rho$</th>
<th>$R_1$</th>
<th>$\tau_{11}$</th>
<th>MOR$^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kgm$^{-3}$</td>
<td>Jm$^{-2}$</td>
<td>kPa</td>
<td>MPa</td>
</tr>
<tr>
<td>A</td>
<td>520</td>
<td>1295.33</td>
<td>20861</td>
<td>41.6</td>
</tr>
</tbody>
</table>

$\rho$ – density, MOR – modulus of rupture in bending (* values were taken from Krzosek (2009))
In case of circular sawing in which indirect positions of the cutting edge are present, \( R \) and \( \tau \) have to be calculated from formulae (7). According to Aydin et al. (2007) it was assumed that fracture toughness for pine for longitudinal (axial) cutting \( R_{||} = 0.05 R_{\perp} \). Moreover, an assumption was made that in the case of pine wood for axial cutting the shear yield stress \( \tau_{||} \) is equal to 0.125 MOR (modulus of rupture in bending (Kretschmann 2010, Krzysik 1974)). The set of the raw material is presented in tab. 2. Values of \( R_{\perp} \) and \( \tau_{\perp} \) were determined during sawing tests according to the methodology described in works by Orlowski and Atkins (2007). It should be also emphasised that in the cutting tests of which results are in Table 2 the same samples have been re-sawn which earlier had been tested by Krzosek (2009).

**RESULTS AND DISCUSSION**

In Figure 2, the values of the shear yield stresses \( \tau_{||} (\varphi) \) (Fig. 2a) and fracture toughness \( R_{||} (\varphi) \) (Fig. 2b) for indirect positions of the cutting edge of Polish pine wood from the Baltic Natural Forest Region provenance are presented. It could be emphasised that for the cutting edge position \( \varphi = 90^\circ \), while there are conditions of perpendicular cutting, in case of the shear yield stresses the largest differences caused by raw material provenance were observed (Orlowski and Ochrymiuk 2013). On the other hand, for fracture toughness mentioned differences were not so meaningful (Orlowski and Ochrymiuk 2013).

![Figure 2](image-url). The effect of the cutting edge positions in relation to the grains of Polish pine wood from the Baltic Natural Forest Region on shear yield stresses (a) and fracture toughness (b)

Results of predictions of cutting powers obtained with the use of a new developed cutting model FM_V that include work of separation in addition to plasticity and friction in the case of sawing of pine of the Baltic Natural Forest Region provenance with one circular saw blade, at the feed speed \( v_f = 153 \text{ m·min}^{-1} \), for one full revolution of the tool (the first one), is shown in Fig. 3a. In Figure 3b results of computation for the same data with the Axelsson’s model AM_V applied are presented. Results of computations, as RMS values, for one saw blade with a new analytical macro-mechanic model FM_V (described in this paper, at the feed speed \( v_f = 70 \text{ m·min}^{-1} \) (\( h = 0.00032 \text{ m} \)), 110 m·min\(^{-1}\) (\( h = 0.0005 \text{ m} \)) and 153 m·min\(^{-1}\) (\( h = 0.0007 \text{ m} \)) at the depth of
cut \( H_p = 80 \text{ mm} \), and for the same data with the Axelsson’s model AM_V, are shown in Fig. 4. Furthermore, in Figure 4 results of computations carried out, for the mean uncut chip thicknesses for the whole range of the machine tool feed speeds (\( v_f = 60–200 \text{ m\cdot min}^{-1} \), \( h = 0.000273–0.000913 \text{ m} \), with the model FM_A presented by Orlowski et al. (2013) and the Axelsson’s model AM_A are presented.

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**Figure 3.** Comparison of predictions of cutting powers obtained with the use of a new developed cutting model FM_V (a) and the Axelsson’s model AM_V (b) in the case of dry pine sawing with one circular saw blade (at the depth of cut \( H_p = 80 \text{ mm} \), feed speed \( v_f = 153 \text{ m\cdot min}^{-1} \)).

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**Figure 4.** Comparison of predictions of cutting powers obtained with the use of: FM_V model and AM_V (for \( h = 0.00032 \text{ m}, 0.0005 \text{ m}, 0.0007 \text{ m} \)), and the models FM_A and AM_A in which calculations were done for the mean uncut chip thicknesses (for \( h = 0.000273–0.000913 \text{ m} \)) in the case of dry pine sawing with one circular saw blade.

In both cases values of RMS of cutting powers obtained with a new developed model (macro-mechanic) FM_V and AM_V are larger than cutting power values computed with...
the use of the mean uncut chip thicknesses (FM_A and AM_A). Values of cutting powers from the Axelsson’s model AM_A are considerably lower than values from the FM_A model, e.g. for uncut chip thickness of \( h = 0.0009 \) m the difference is 30%. In the industrial experiments of Scot pine wood sawing Cristóvão et al. (2013a, b) also found out that values from the forecasting with the Axelson’s model are fairly lower from measured ones. For climb sawing the difference was about 30%, and for counter sawing the difference was even almost 40%.

It ought to be emphasised that results of cutting power from the FM_A model quite well correspond to the results obtained from the forecasting with the use of the specific cutting resistance \( k \), from the works Manžos (1974) and Orlicz (1988), as it has been proven in the paper by Orlowski et al. (2013). This compatibility relates only for pine wood from the Baltic Natural Forest Region provenance.

**CONCLUSIONS**

The conducted analyses of energetic effects using cutting models that include work of separation in addition to plasticity and friction, in both cases FM_V and FM_A models, revealed that values of cutting powers for cutting pine wood on the circular sawing machine differ significantly from values obtained with models which base on the Axelsson empirical model. The maximum difference of cutting powers for uncut chip thickness of \( h = 0.0009 \) m is 30%. For that reason it is difficult to recommend the latter model for forecasting of power consumption for sawing process with circular saw blades.

In each case values of \( RMS \) of cutting powers obtained with a new developed model FM_V are larger than values computed with the use of the mean uncut chip thicknesses in the model.

The presented model FM_V provide general information about the relations between the energetic effects, raw material data and the process parameters. In addition, it can be used to simulate real sawing pattern cases on circular sawing machines, and the best set of parameters to improve the process performance can be selected.

revealed the usefulness for every known type of sawing kinematics.

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