ASSESSMENT OF SUITABILITY OF THE FRACTAL DIMENSION FOR DESCRIBING THE ROUGHNESS OF SURFACES OF WOODEN MATERIALS

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Abstract

In this contribution the theoretical bases and the possibilities of application of the fractal dimension are presented as a complementary index for assessing the roughness of surfaces. The results of experimental research concerning the evaluation of the fractal dimension for the roughness of surfaces of samples of spruce and oak wood being subject to milling with high machining speeds are discussed. The suitability of the fractal dimension for describing the state of geometrical structure of the surface is assessed.

Key words: *fractal dimension, wood, geometrical structure, measurements*

1. INTRODUCTION

Examination of the description of geometrical structure of surfaces shows that besides kinematic-geometric mapping there are also disturbances the source of which has to be found out in the dynamic behavior of the "machine tool – fixture – workpiece – cutting tool" system during machining. The identification of these disturbances requires applying non-conventional techniques of information gathering, as ex.: fuzzy logic, disaster theory, chaos theory and fractals being more and more used in different domains of life. In the case of examination of the geometrical structure of surfaces, fractals seem to be a very useful as well as convenient tool. They allow a compact description of complexity of models appearing in practical problems, with the use of a small number of parameters [2,5,6]. At the same time, it is an essential fact that the way for describing chaos of the system concentrates on analyzing its model during functioning, and not on searching for a mathematical formula that accurately describes the model being examined [1].

2. FRACTAL DIMENSION AND ITS PROPERTIES

From the mathematical point of view a fractal is a complex mathematical figure about which it can be said with difficulty whether it is a curve, a surface, or whether it has a greater dimension; it is characterized by a specific regularity of irregularity – the degree of that regularity is determined by a non-integer number – the fractal dimension. The dimension is a property of geometrical objects. It can be agreed that the most essential

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explication of that notion consists therein that it is a number of independent parameters being necessary for describing univocally the points of a given object. Unfortunately, the development of mathematics has shown that one cannot describe all known objects by means of only one notion. Thus, at present the word „dimension" has many meanings; for instance, one can consider topological dimension, Hausdorff dimension, fractal dimension, self-similarity dimension, box dimension, capacity dimension, information dimension and many others. Sometimes all these dimensions are suitable, and can even be mutually equal, and sometimes they can mutually be contradictory. In the case of geometry it is reasonable to use the self-similarity dimension, the circle dimension and the box dimension. These are all particular cases of the fractal dimension which characterizes an object being subject to homogeneous scaling in space; it is generally determined in the phase space of the system. The above mentioned fractal dimension forms are rather difficult to be calculated even for comparatively simple sets; therefore, in calculating practice the box dimension is used – named also Kolmogorov dimension. This dimension enables to perform systematic measuring which can be applied to an arbitrary structure on the plane, and it can be easily adapted to structures appearing in three-dimensional space [3].

Establishing the Kolmogorov fractal dimension consists in placing the being examined structure on a regular net with mesh size s and calculating the "boxes" of the net in which a fragment of the structure is included. In this way the number N is obtained that depends upon the size of meshes s. This dependence is indicated as to be $N(s)$. Then the value s is gradually diminished and the corresponding numbers $N(s)$ are found. The following step consists in performing the diagram of logarithms in the dependence $\log(N(s))$ in function $\log(1/s)$. The inclination of the straight line resulting from the connection of the particular points on the diagram enables to read the box dimension D. In Fig. 1 the illustration of the given measuring procedure for two points is presented. In that case, the box dimension $D = 1.45$ [5].
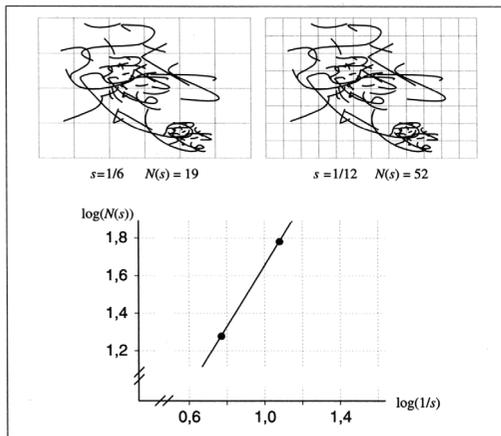

$s=1/6$　　$N(s) = 19$　　　$s = 1/12$　　$N(s) = 52$

Fig. 1. Reckoning the boxes for the determined structure [5]

For real surfaces (3D) the fractal dimension will attain values equal to $2 < D < 3$, where if $D \longrightarrow 2$, one has to do with a simple and organized system of geometrical structure of the surface. When $D \longrightarrow 3$, the being considered surfaces are characterized by a chaotic random system of geometrical structure. In Fig. 2 surfaces are shown, their fractal dimensions and fractal diagrams $\log n(\varepsilon)/\log(1/\varepsilon)$ [5].
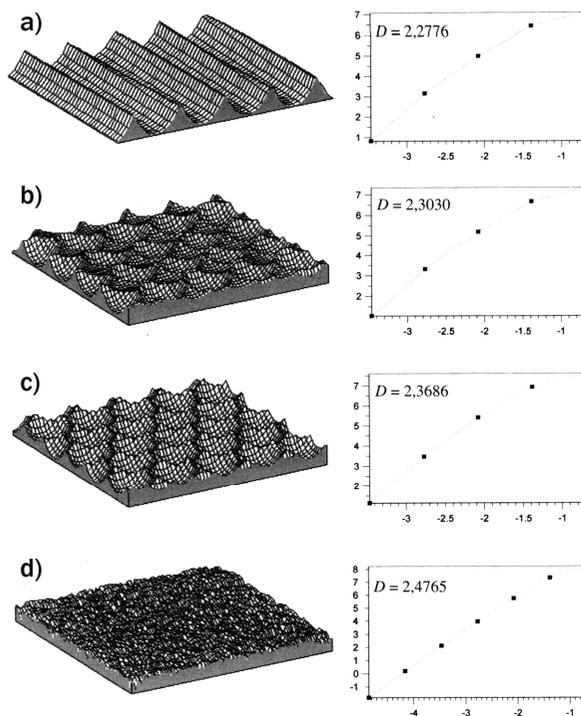
Fig. 2. Fractal dimensions of surfaces with various degrees of complexity: anisotropic with symmetry multiplication factor a) L^2, b) L^4 and c) L^6 and d) random isotropic, where L – axis of symmetric transformations [4]

3. METHODOLOGY OF EXPERIMENTAL RESEARCH

The research was carried on for two species of wood being generally used in the furniture industry as well as for the production of elements of building woodwork. The first among them is oak wood (deciduous tree), and the second spruce wood (coniferous tree). From both selected species two standard cubicoids were prepared with humidity 10 ± 2 %. For machining the samples the cutterhead of the Benmet Company (the Czech Republic) was applied. The diameter of the tool was 18 mm. There was one cutter of Pilana (the Czech Republic) in the cutterhead; the cutter was executed of high-speed steel, $100 \times 30 \times 3$ mm, $\beta = 40°$). For the research, four different speed rotations were selected of the milling cutter with diameter 18 mm: $n = 10000, 13000, 16000, 19000$ rev/min. For all the samples the selected rate of travel was $vf = 4$m/min.

4. ROUGHNESS MEASURING STAND

The measurements of roughness were carried out on a profilographmeter Form Talysurf 120L of the Taylor Hobson Limited. The functioning of the profilographmeter is controlled by a computer Dell OptiPlex GX110. The computer is equipped with a Pentium III processor and RAM 128 MB memory. The max length of displacement of the needle

(length of the being measured profile) is 120 mm. The deviation from the mean height can amount to ±3 mm. In Fig. 3 the scheme of the measurung stand is presented.

Rys.3. Profilographmeter FormTalysurf 120L

The measurements of the 3D roughness were carried out on a measuring surface with dimensions 1.28 x 1.28 mm, with a line step of 5 μm. The so measured surfaces were filtered in order to separate roughness from waviness and shape errors by a Gauss filter with "cut-off" value of 0.25 mm. The modeled surfaces were seized in tables and transferred to the Talymap Export 2.0.19 program that serves the profilometer applied for measurements and analysis of real surfaces. Exemplary pictures of surfaces, obtained as a result of performing a 3D measurement are presented in Fig. 4.
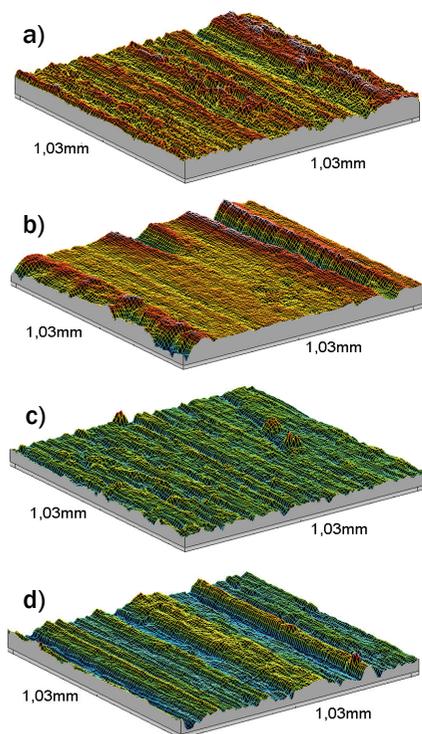

Fig. 4. Exemplary pictures of 3D surfaces:
a) oak sample n=10000 rev/min,
b) oak sample n=19000 rev/min,
c) spruce sample n=10000 rev/min,
d) spruce sample n=19000 rev/min.

5. ANALYSIS OF RESEARCH RESULTS

The results of measurement of roughness 3D parameters as well of the Kolmogorov dimension, defined for oak and spruce wood subject to milling at high machining speed, are presented in Table 1. The dimension can be calculated by approximating by a straight line the diagrams in log n(ε)/ log(1/ε) co-ordinates. On the basis of the angle of inclination of the approximating straight line the numerical value of the fractal dimension of Kolmogorov was determined.

Table 1. Roughness 3D parameters and fractal dimension *D* of the milling oak and spruce sample surface (*n* [rev/min] and *r* – correlation coefficient)

	Oak sample					Spruce sample				
n	10000	13000	16000	19000	*r*	10000	13000	16000	19000	*R*
D	2.58	2.55	2.5	2.45		2.6	2.53	2.5	2.46	
	Amplitude parameters of roughness									
Sa	1.77	0.826	2.28	2.5	-0.71	2.06	1.62	2.42	2.3	-0.61
Sq	2.3	1.13	3.56	3.68	-0.76	2.87	4.15	6.21	5.26	-0.79
Sz	16.7	9.34	32.2	28.8	-0.75	19	20.1	27	72.3	-0.89
Ssk	-0.176	-1.74	-0.993	0.251	-0.39	0.114	-0.479	-0.438	0.82	-0.53
Sku	3.55	7.3	9.78	16.02	-0.98	2.82	4.4	5.73	15.4	-0.91
Sp	8.4	4.1	16.9	30.7	-0.92	13.6	15.5	17.3	18.6	-0.98
Sv	11	17	23.7	15.2	-0.41	14.4	16.8	16.6	15.96	-0.43
St	19.4	21.1	40.6	45.9	-0.96	28.0	32.3	33.9	34.56	-0.91
	Spatial parameters of roughness									
Str	0.0555	0.0252	0.0479	0.0557	-0.31	0.639	0.073	0.9481	0.0655	-0.22
Std	-0.0143	0.0373	0.0212	0.0045	-0.12	0.0089	0.0886	0.0925	0.00098	0.15
Sal	0.0339	0.0153	0.0255	0.0304	-0.96	0.0324	0.0273	0.023	0.018	0.99
Sds	531	634	452	394	0.79	397	545	494	467	0.83
	Hybrid parameters of roughness									
Sdq	0.161	0.106	0.198	0.204	-0.69	0.441	0.178	0.225	0.171	0.71
Ssc	0.0168	0.018	0.0255	0.0266	-0.96	0.0396	0.02	0.0225	0.0154	0.93
Sdr	1.27	0.558	1.88	2	-0.74	8	1.56	2.45	1.43	0.71

When analyzing the results collected in Table 1 it can be observed that the Kolmogorov fractal (box) dimension correlates to different degree with roughness D3 parameters. The correlation coefficient *r* between the fractal dimension and the roughness parameters in case of both wood species attained the highest values, above 0.90, for the following amplitude parameters: *Sku*, *Sp*, *St*, as well as for the spatial parameter *Sal* and for the hybrid parameter Ssc.

Thus, what concerns the results obtained during the performed research it can be stated that changes of the fractal box dimension can argue changes of the following features appearing on the examined surface: relative measure of concentration and flatting of tips of roughness (i.e. kurtosis of tips), height of roughness tips, total roughness height, length of the segment of the most rapid fading of autocorrelation function – determining the part of roughness and waviness on the examined surface, and arthmetic mean of curvatures of tips of surface uneveness.

It appeared that the fractal dimension was least of all sensitive on directionality of the surface structure being described by parameter *Std* and on changes of index of texture *Str*. The obtained values of parameter *Str* argue an invariable, for all the being examined structures,and very high degree of anisotropy of surfaces of both examined wood species.

6. SUMMARY

The development of methods of measuring surface roughness makes it possible to present the results in the form of anisotropic pictures. They enable to obtain a more precise description of geometrical features and other properties obtained during surface shaping. However, even spatial pictures of surfaces and a rich set of parameters describing their geometrical features are unsatisfactory for characterizing the degree of arrangement of the structure appearing on the examined surfaces after various kinds of treatment/machining.

In this contribution, the possiblity is presented of applying fractal analysis as an alternative tool assisting the description of the geometrical state of the examined surface. During the research carried out a diminution of the fractal dimension in case of increasing rotational speed of the milling cutter machining both oak wood and spruce wood was observed. That is a proof of diminution of the disordered state of the being created surface structures when the speed of machining is increasing. In the case of surfaces machined with the lowest rotations of the cutting tool $n = 10000$ rev/min, the examined Kolmogorov dimension attained values of 2.58 i 2.6 – and for rotations of the tool $n = 19000$ rev/min, respectively – 2.45 and 2.46. It is possible to utilize the existing correlation between the fractal (box) Kolmogorov dimension and the degree of randomness of geometrical structures obtained on surfaces of wood machined by various methods, for completing the description of qualitative features of surfaces.

7. LITERATURE

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