



## CONVENTIONAL METHODS AND NEW APPROACH TO DETERMINE CUTTING RESISTANCE

Lud'ka Hlásková – Zdeněk Kopecký – Michal Jedinák

### Abstract

*This paper presents new calculating model using the application of fracture mechanics to determination cutting resistance. In the conventional methods, energetic effects (cutting forces, cutting power and cutting resistance) of wood sawing process are generally calculated on the basis of the specific cutting resistance, which is in the case of wood cutting the function of many factors. New model uses to determination cutting, feed, shear and friction forces the application of the Ernst-Merchant theory in the conditions of circular-saw blade cutting. It includes the prediction of the shear plane angle for the cutting models, which include fracture toughness in addition to plasticity and friction, broaden possibilities of energetic effects modelling of the sawing process even for small values of the uncut chip.*

**Key words:** *Cutting resistance, circular-saw blade, fracture mechanics, shear yield stress, fracture toughness, native beech*

### INTRODUCTION

In the wood processing industry, starting with primary production up to the machining of the final product, the circular-saw blade cutting is the most frequent way to machine materials on the basis of wood, plastic, as well as composite materials. Despite the relatively extensive theoretical and practical knowledge of wood machining, no process is currently known which would help to accurately determine the magnitude of cutting resistance and cutting force. It is relatively difficult to determine individual components of cutting resistance which depend on many other factors. The demanding nature of the search for the optimum results is significantly influenced by wood properties, by its anisotropy and variations of physical and mechanical properties in relation to the direction of grains. Nowadays, different modifications of two basic methods are used for theoretical purposes and in practice – the technological and physical method and analytical method (Lisičan 1996).

According to the latest theoretical findings with the use of fracture mechanics methods (Atkins 2003, 2009) and (Orlowski 2010, Orlowski et al. 2012), a mathematical model of power when cutting by saw blades can be expressed in the following form

$$\bar{P}_{cw} = F_c \cdot v_c + P_{ac} = \left[ z_a \cdot \frac{\tau_\gamma \cdot b \cdot \gamma}{Q_{shear}} \cdot h_m \cdot v_c + z_a \cdot \frac{R \cdot b}{Q_{shear}} \cdot v_c \right] + \dot{m} \cdot v_c^2 \quad [\text{W}] \quad (1)$$

The first equation member expresses the power necessary for bending and subsequent removal of the chip, the second member expresses the power for overcoming friction between the workpiece and the tool edge, including the formation of a new surface, and the third member expresses the power necessary for the chip acceleration and its sweep out of the point of cutting. However, the third member does not express force ratios at the chip separation (no effect on cutting resistance), but expresses kinetic energy for carrying chips (sawing) out of the cut by the saw blade. This means that it only affects the total consumed saw power (Orlowski et. al 2012). The following is applied for the mass flow of chips:

$$\dot{m} = \frac{b \cdot l \cdot v_f \cdot \rho}{2} \quad [\text{kg} \cdot \text{s}^{-1}] \quad (2)$$

Under the theory which uses fracture mechanics, the cutting force, related to one blade tooth, is expressed by the slope of the line in the form  $y=(k) \cdot x+(q)$  (Orlowski and Palubicki 2009, Orlowski 2010)

$$F_c^{1z} = \left( \frac{\tau_\gamma \cdot b \cdot \gamma}{Q_{shear}} \right) \cdot h_m + \left( \frac{R \cdot b}{Q_{shear}} \right) \quad [\text{N}] \quad (3)$$

where:  $\tau_\gamma$  is shear yield stress (Pa),  $R$  is specific work of a surface separation (fracture toughness) ( $\text{J} \cdot \text{m}^{-2}$ ),  $b$  is the width of a saw kerf,  $Q_{shear}$  is a friction correction coefficient (-),  $\gamma$  shearing strain along the shear plane (-).

Shearing strain along the shear plane is possible to obtain from the formula

$$\gamma = \frac{\cos \gamma_f}{\cos(\Phi_s - \gamma_f) \cdot \sin \Phi_s} \quad [-] \quad (4)$$

where:  $\gamma_f$  is tooth rake angle,  $\Phi_s$  is shear angle, which expresses the orientation of the shear plane in relation to the worked surface, and which is calculated with the use of the Ernst-Merchant diagram (Fig. 1).

$$\Phi_s = \left( \frac{\pi}{4} \right) - \left( \frac{1}{2} \right) \cdot (\Theta_\mu - \gamma_f) \quad [^\circ] \quad (5)$$

where:  $\Theta_\mu$  friction angle obtained from  $\tan^{-1} \mu = \Theta_\mu$  ( $\mu$  is friction coefficient),  $\pi$  [rad] 180°

The friction correction coefficient  $Q_{shear}$  depends substantially on the orientation of the shear plane towards the worked surface. When shear angle  $\Phi_s$  equals zero (the tool cuts off no chips), the friction correction coefficient  $Q_{shear}$  equals one. (Orlowski and Palubicki 2009, Orlowski 2010).

$$Q_{shear} = 1 - \frac{\sin \Theta_\mu \cdot \sin \Phi_s}{\cos(\Theta_\mu - \gamma_f) \cdot \cos(\Phi_s - \gamma_f)} \quad [-] \quad (6)$$

Cutting and feed forces are determined by the application of the Ernst-Merchant theory (Atkins 2003) in the conditions of circular-saw blade cutting. The chip formation process is the main factor for the determination process of cutting and feed forces.

**Forces:**
 $F_a$  = activ force

 $F_c$  = cutting force

 $F_{cN}$  = normal to cutting force

 $F_f$  = feed force

 $F_{fN}$  = normal to feed force

 $F_s$  = shear force

 $F_{sN}$  = normal to shear plane

 $F_\gamma$  = friction force

 $F_{\gamma N}$  = normal to friction force

**Angles:**
 $\gamma_f$  = rake angle

 $\Theta_\mu$  = friction angle

 $\Phi_s$  = shear angle

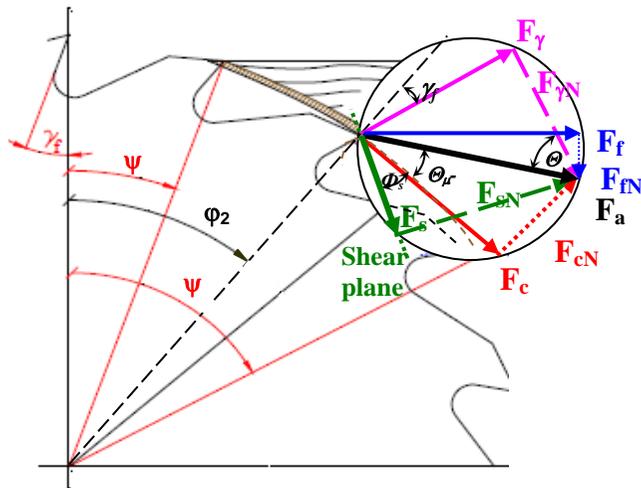
 $\varphi_2$  = angle between the cutting plane and the direction of grains


Fig. 1 Simplified cutting process model with Ernst and Merchant's force circle

The circular-saw blade cutting process is not an example of purely orthogonal cutting (angle between the direction of grains and cutting velocity vector may differ by up to  $90^\circ$  ( $\varphi_3 = 0-90^\circ$ ). Hence, taking into account the position of the cutting edge in relation to the grains, for indirect positions of the cutting edge fracture toughness  $R$  and the shear yield stress  $\tau_\gamma$  may be calculated from formulae known from strength of materials (Orlicz 1988). For example for cutting on circular sawing machines these material features are as follows:

$$R_{\parallel\perp} = R_{\parallel} \cos^2 \varphi_2 + R_{\perp} \sin^2 \varphi_2 \quad (7)$$

$$\tau_{\gamma\parallel\perp} = \tau_{\gamma\parallel} \cos^2 \varphi_2 + \tau_{\gamma\perp} \sin^2 \varphi_2 \quad (8)$$

$\varphi_2$  angle between the cutting plane and the direction of grains (Fig. 1)

## MATERIAL AND METHODS

The experiment was performed on a modernized experimental testing device used for research of circular sawblade cutting [Kopecký, Rousek 2012]. This device simulates the conditions of circular-saw blade cutting in the real operation as precisely as possible. The parameters of the cutting process (cutting force  $F_c$ , feed force  $F_f$  and workpiece feed velocity  $v_f$ ) were recorded by sensors installed in the measuring stand. The signals from the sensors were transferred in the data switchboard Spider 8 and in the software Conmes Spider and subsequently processed into tables and graphs.

The cutting process was performed with a saw blade manufactured by Flury Systems AG (Fig.1). It is a standard blade with the diameter of 350 mm with straight tooth designed for longitudinal cutting of wood. The construction parameters of the blade are shown in Tab. 1. The cutting was performed under the optimum operation speed  $n = 3800 \text{ min}^{-1}$ , i.e. under cutting velocity  $v_c = 70 \text{ m}\cdot\text{s}^{-1}$ . Workpiece feed velocity varied within the range of  $v_f = 2, 4, 6, 8, 10, 13, 16, 19$  and  $22 \text{ m}\cdot\text{min}^{-1}$ . This corresponded with the changing feed per tooth  $f_z$  and mean chip thickness  $h_m$ .

Table 1 Parameters of saw blade K3

Parameters of saw blade K3 – Flury systems 350	
Saw blade diameter D	350 mm
Number of teeth z	28
Clamping hole diameter d	30 mm
Blade body width s	2,5 mm
Tooth width (cutting joint) $s_t$	3,5 mm
Tooth height h	10,5 mm
Tooth pitch $t_p$	39,27 mm
Clearance angle $\alpha_f$	15°
Cutting edge angle $\beta_f$	55°
Rake angle $\gamma_f$	20°

In order to verify the validity and function of the new calculation model, the samples of native beech ( $\rho = 649 \text{ kg}\cdot\text{m}^{-3}$ ) were used in the experiment. The scantlings ( $700 \times 120 \times 22 \text{ mm}$ ) were dried (relative moisture content 7,5 %) and unified in the thickness  $e = 22 \text{ mm}$ . Five measurements were performed, which were statistically evaluated.

## RESULTS AND DISCUSSION

The validity verification of the new calculation model was performed by an experiment. Fig. 2 shows the relation of cutting force which was obtained by experimental measurement as a function of mean chip thickness. Almost linear increase of cutting force occurred along with the growing chip thickness, which confirms the theoretical assumptions, see equation 3. Measured data were statistically evaluated – the standard deviation is in the range  $s = 0.39 - 0.95 \text{ N}$ .

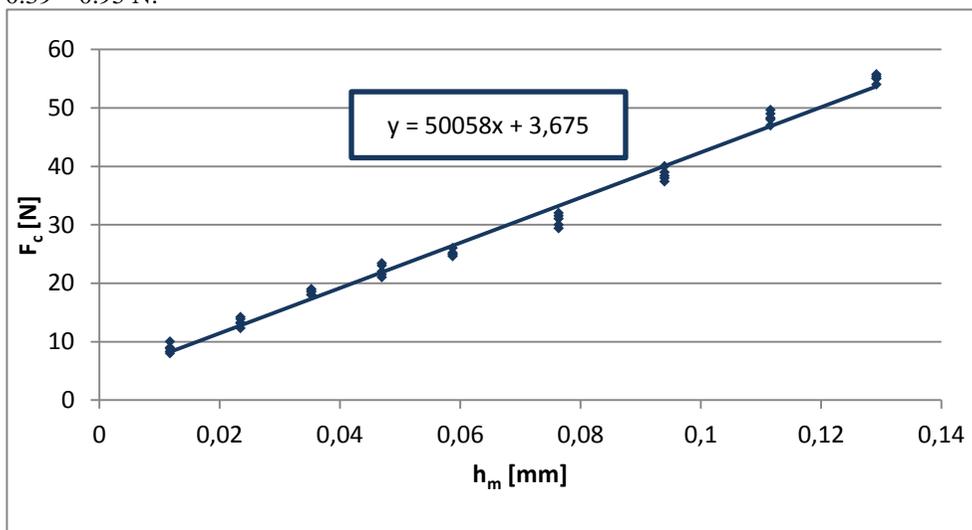


Fig. 2 Cutting force as a function of mean chip thickness

The determination of the main parameters of the model is based on the regression analysis. The fracture toughness  $R_{\perp}$  (for  $\varphi_2 = 38.68^\circ$ , Fig. 1) was determined from the line sift and shear yield stress  $\tau_{\gamma\perp}$  from its slope (Atkins 2005, Orłowski and Palubicki 2009, Orłowski 2010). The application of experimental data in the proposed model brings significant data for the longitudinally transversal cutting model to the circular saw blade cutting process, see (Tab. 2):

Table 2: Results obtained by the application of fracture mechanics

	$\rho$ ( $\text{kg}\cdot\text{m}^{-3}$ )	$\tau_{\gamma\perp}$ (MPa)	$R_{\perp}$ ( $\text{Jm}^{-2}$ )	$\mu$	$\Theta_\mu$	$Q_{\text{shear}}$
Beech	649	42.915	1020.833	0.97	43,99	0.576

The Ernst-Merchant theory (equation 5) helped to calculate the orientation of the shear plane in respect to the worked surface  $\Phi_s = 33^\circ$  and the shearing strain along the shear plane  $\gamma = 1.87$  for native beech under the longitudinal transverse cutting model. These values are the input data when calculating the specific cutting resistance.

Specific cutting resistance which was calculated using the fracture mechanics was compared to the conventional methods of calculation which are based in case of statistical and physical method on statistical evaluation of many times performed experiments (Lisičan, 1996):

$$k_c = \frac{(1,1 \div 1,2) \cdot k_{\varphi_2\delta}^1 \cdot k_d \cdot k_v \cdot k_w \cdot k_\rho \cdot k_t}{h^m} + \frac{\xi \cdot e}{b} \quad [\text{Pa}] \quad (9)$$

and in case of analytical method on research according Lisičan:

$$k_c = k_{\leq 0,1}^1 + \frac{(a_\rho - 0,8) \cdot p}{h} + \frac{\xi \cdot e}{b} \quad [\text{Pa}] \quad (10)$$

The Atkinson model, which includes the fracture toughness  $R$  (equation 1), can help to derive a relationship for the calculation of specific cutting resistance  $k_c$ :

$$k_c = \frac{1}{Q_{\text{shear}}} \left( \tau_\gamma \cdot \gamma + \frac{R}{h} \right) \quad [\text{Pa}] \quad (11)$$

The formula for the calculation of specific cutting resistance shows that the specific cutting resistance will increase sharply with a small feed per tooth (with small chip thickness  $h$ ).

Another Figure (Fig. 3) shows modelling of the functional relationship of the specific cutting resistance and chip thickness. The calculations were performed for the chip thickness of  $h = 0.012 - 0.14$  [mm].

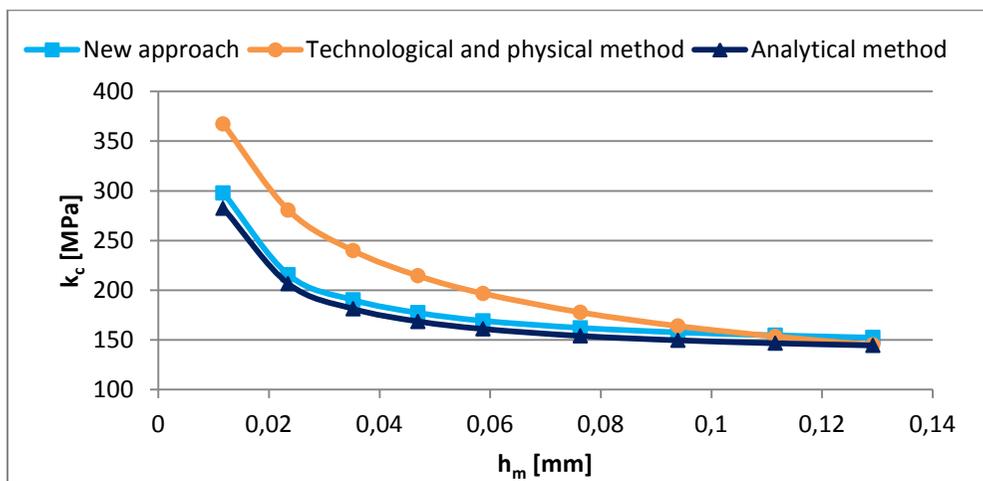


Fig. 3 Relationship of specific cutting resistance and mean chip thickness

Technological and physical method quite differs from results which were obtained by experiment and calculated using new model. This is probably caused by the fact that the conventional method is based on a large amount of the correction factors (equation 9) which are determined statistically from a large number of experiments and therefore can not be exactly accurate.

Analytical method belongs to highly accurate method of calculation specific cutting resistance therefore we can see negligible differences between new approach results and results obtained by analytical method. As well as it is necessary to say that this conventional method is relatively old and calculation is complicated therefore there is a need to design new calculation model.

Comparing the results of conventional methods and a new approach it is obvious that good accordance occurs only in higher thickness of chip which is not typical for cutting by circular saw blades. In the limit thickness of chip (10-5  $\mu\text{m}$ ) for format circular saw it is already a noticeable difference in the accuracy of the proposed model (verified by experiment) and the conventional methods.

We can also see that the specific cutting resistance decreases with the increasing chip thickness. This phenomenon is known from metal machining. In contrast, under very small feeds per tooth when chip thickness comes closer to the existing cutting edge radius, the hyperbolic increase in the specific cutting resistance  $k_c$  occurs.

## CONCLUSION

Despite the fact that the circular-saw blade cutting process is not an example of purely orthogonal cutting, the application of results obtained from the experimental measurement results allowed us to determine the fracture toughness and shear yield stress for longitudinal transversal model of cutting beech by a saw blade. Knowing these two parameters, it is possible to make prognosis for the necessary cutting power and forces affecting the workpiece and the tool.

### Acknowledgement

This paper was supported by the European Social Fund and the state budget of the Czech Republic, project "The Establishment of an International Research Team for the Development of New Wood-based Materials" reg. no. CZ.1.07/2.3.00/20.0269 and „Postdoctoral positions in technical and economic fields on MENDELU“ reg. No. CZ.1.07/2.3.00/30.003. The author thanks for a financial support to deal with the projects.

### REFERENCES

1. ATKINS A.G., 2003: Modelling metal cutting using modern ductile fracture mechanics: quantitative explanations for some longstanding problems. *International Journal of Mechanical Sciences*, 45: 373-396.
2. ATKINS A.G., 2009: The science and engineering of cutting. The mechanics and process of separating, scratching, and puncturing biomaterials, metals and non metals. Butterworth-Heineman is an imprint of Elsevier, Oxford, 2009, 413 p.
3. KOPECKÝ Z., ROUSEK M., 2012: Impact of dominant vibrations on noise level of dimension circular sawblades. *Wood Research*. 2012. Vol. 57, No. 1, pp. 151-160. ISSN 1336-4561.
4. LISIČAN J. a kol., 1996: Theory and wood technology. Zvolen: Matcentrum. 626 p.
5. ORLICZ T., 1988: Wood machining with cutting tools. Warsaw University of Life Science.
6. ORLOWSKI K., PALUBICKI B., 2009: Recent progress in research on the cutting processes of wood. A review COST Action E35 2004–2008: Wood machining – micromechanics and fracture. *Holzforschung*, Vol. 63, iss. 2: 181–185. ISSN 0018-3830.
7. ORLOWSKI K., 2010: The fundamentals of narrow-kerf sawing: mechanics and quality of cutting, Technical University in Zvolen, pp. 1-123, ISBN 978-80-228-2140-7.
8. ORLOWSKI K., OCHRYMIUK T., ATKINS T., CHUCHALA D., 2012: Application of Fracture Mechanics for Energetic Effects Predictions While Wood Sawing. *Wood Science and Technology*, Vol. 43., No 3/2013. ISSN 0043-7719.